Homework 7

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**Problem 12**

1. If a­n = 0 for all n ≥ 2 then -3(0) + 4(0) = 0 = a­n
2. If a­n = 1 for all n ≥ 2 then -3(1) + 4(1) = 1 = a­n
3. If a­n = (-4)n for all n ≥ 2 then -3(-4)n-1 + 4(-4)n-2 = (-4)n[-3(-4)-1+4(-4)-2] = (-4)n[1] = (-4)n = a­n
4. If a­n = 2(-4)n+3 for all n ≥ 2 then -3[2(-4)n-1+3]+4[2(-4)n-2+3] = -6(-4)n-1-9+8(-4)n-2+12 = (-4)n[-6(-4)-1+8(-4)-2]+3 = 2(-4)n+3 = an

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**Problem 26(a)**

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| a1 | a2 | a3 | a4 | a5 | a6 | a7 | a8 | a9 | a10 |
| 3 | 6 | 11 | 18 | 27 | 38 | 51 | 66 | 83 | 102 |
| 3 | 3+3 | 6+5 | 11+7 | 18+9 | 27+11 | 38+13 | 51+15 | 66+17 | 83+19 |
| 3 | a1+3 | a2+5 | … | … | … | … | … | … | a9+19 |
| 3 | a1+(2\*2-1) | a2+(2\*3-1) | … | … | … | … | … | … | a9+(2\*10-1) |

an =an-1+(2n-1), a11 = 123, a12 = 146, a13 = 171

**Problem 26(c)**

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| a1 | a2 | a3 | a4 | a5 | a6 | a7 | a8 | a9 | a10 | a11 |
| 1 | 10 | 11 | 100 | 101 | 110 | 111 | 1000 | 1001 | 1010 | 1011 | BINARY |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | DECIMAL |

an = n in binary, a12 = 1100, a13 = 1101, a14 = 1110

**Problem 26(g)**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| a1 | a2 | | a3 | | | a4 | | | | a5 | | | | |

For all even n: an = n\*{0}

For all odd n: an = n\*{1}

a6 = 6\*{0} = {0,0,0,0,0,0}

* Next 3 terms: 0,0,0

**Problem 32**

**Problem 34**

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**Problem 4**

1. Integers not divisible by 3: 3n+1or 3n+2. This can be expressed as This is a subset of Z+ hence it is countable.
2. Integers divisible by 5 and not by 7: 35n+1, 35n+2, 35n+3, 35n+4, 35n+5, 35n+6. There is a 1-1 correspondence to Z+ such that f(6n) = 35n+1, f(6n+1) = 35n+2, … , f(6n+5) = 35n+6. This is countably infinite.
3. Let A be the set containing numbers with decimal representations consisting of all 1’s. For i in A, let y = numbers of 1’s to the left of the decimal, let z = number of 1’s to the right of the decimal. Then map to f(x) = 2y\*3z. f(x) is now countable.
4. Let f be a function that replaces all 9's by zeros, then its range is all the binary representations of real numbers. Since all the real numbers can be represented in base 2, this group is not countable.